Lab Assignment #8

Sampling Theorem of Spectral Functions

By: Prof. Dr. rer. nat. habil. Albrecht Rost
University of Applied Sciences Merseburg
Department 1: Computer Science and Applied Natural Sciences
Physics, Laboratory of Measurement Technology
e-mail: albrecht.rost@in.fh-merseburg.de

Objective

To study the sampling theorem of spectral functions using Agilent VEE.

Equipment

- Agilent VEE software

A. Theoretical Introduction

The complex transfer function $\underline{g}(\omega)$ of a circuit can be determined by comparing of the spectral function of a time function, the input to the network with the response function the circuit's output. For theoretical considerations usually simple single time functions for example a step or a single pulse function are used. But for measurements it is easier to use periodic instead of single time functions. The question is how conditions must be set up so that these two different methods lead to the same result. The answer is given from the sampling theorem of spectral functions:

> If a single time function F(t) is limited into the time interval $0 \leq t < T$, it's complex spectral function $\underline{c}(\omega)$ is completely determined by the amplitudes of the complex Fourier transformation $\underline{c}_n(n\omega_0)$ of the periodic function $F_p(t)$ with the same shape and with the period $T_0 > T$. The Fourier coefficients $\underline{c}_n(n\omega_0)$ including a fit factor agree with the value of the spectral function $\underline{c}(\omega)$ at these frequencies $n\omega_0 = 2\pi n / T_0$, and the spectral function $\underline{c}(\omega)$ can be calculated from the values $\underline{c}_n(n\omega_0)$ using a well-defined interpolation function.

The sampling theorem of spectral functions is related to another fundamental law of telecommunication. Looking to the spectrum of a single rectangular pulse one can say that the mean part of the spectral power density is inside a frequency range below the n^{th} zero point of the spectral function, where n is a small integer number. That means the higher the bandwidth of a signal, the shorter its duration. This law can be written in the simple equation

$$t_p \cdot B = const,$$

where t_p is the duration and B is the bandwidth of the signal.

B. Simulation Experiments

The sampling theorem of spectral functions is a fundamental law for all transmission measurements, and the conclusions are very important for many practical applications. The following simulation programs illustrate this fundamental law. To run this programs you need the run-time version of Agilent VEE (Version VEE Pro).

1. Program sampling5.vxe

The first simulation demonstrates the agreement between the spectra of a single and a periodic time function with quite the same pulse shape.

Two different time signals are generated and displayed simultaneously: The single time function and the time function generated as a periodic repetition of the single signal. Note that generation of a single time function means that only one pulse appears during the measurement time t_{meas}.

From both signals the spectral functions are calculated applying a Fast Fourier Transformation. Using a fit factor T_0 / t_{meas}, where T_0 is the period of the periodic time function $F_p(t)$ and t_{meas} is the measurement time (time window), this program shows that the amplitudes $\underline{c}_n(n\omega_0)$ of the spectral lines of the periodic function agree with the amplitude values of the spectral function $\underline{c}(\omega)$ of the single time function at these frequencies $n\omega_0$.

Change the frequency of the periodic signal and the signal shape and compare the different results. Notice of the agreement between the both spectra, which is perfect at the frequencies $n\omega_0$. Changing the pulse shape you can see, that the sampling theorem is also correct for different signals.

2. Program sampling6.vxe

This program demonstrates the fundamental law of telecommunication. Also in this program two different time signals are generated and displayed simultaneously and the spectra will be calculated.

Change the frequency of the periodic signal and the signal shape and compare the different results. Read out the frequency of the first or second zero point of the spectral function and multiply this value with the duration t_p of the single pulse (Attention: Different frequency scales are used and the pulse width t_p will be measured at 50% of the maximum value of pulse amplitude). Compare the results for different pulse shapes. Notice of the fact that the first zero point of the spectral function is independent of the periodicity of the signal but dependent of the duration and shape of the single pulse.